

# q-Gamow States as continuous linear functionals on analytical test functions

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## Abstract

We define here q-Gamow states corresponding to Tsallis' q-statistics. We compute for them their norm, mean energy value and the q-analogue of the Breit-Wigner distribution (a q-Breit-Wigner).

**Keywords:** Gamow States, q-Gamow States, Ultradistributions.

# 1 Introduction

In three previous papers [1, 2, 3] we have shown that Gamow-states [4] can be interpreted as Sebastiao e Silva's Ultradistributions [5, 6, 7], whose proper treatment appeals to Rigged Hilbert Space [8, 9, 10].

Lately, one finds many high energy experiments that can be interpreted via Tsallis' q-statistics [11]. Indeed, there has been increased use by LHC experiments of such q-statistics and, specially, of the distribution associated with a stationary state within q-statistics, that seems to describe very well the transverse momentum distributions of all different types of hadrons. All four LHC experiments have published results for these distributions that are well fitted by the q-exponential function. The resulting value of q is around 1.15, quite different from Shannon's-Boltzmann's  $q = 1$ . This means that the stationary states of the particles before the hadronization are not in thermal equilibrium. Moreover, the distribution is very robust and practically the same for different hadrons, spanning a range of different energies. Maybe, one of the most impressive results, recently published, is the measurement of the  $p_T$  distribution over a logarithmic range of 14 decades. It was found that the same expression of a q exponential ( $q = 1.15$ ) fits the data over the full range of these fourteen decades. A theory that fits a range of couple of decades is already very interesting but doing so with such a large range of decades, with the same distribution, is rare indeed (see, for instance, [12, 13]).

These circumstances strongly motivate us to investigate complex energy states related to the q-exponential distributions, that is, q-Gamow states, and establish their relation with Gamow-states. Our focus attention then on decay states at a great distance from the dispersion center and ascertain that a q-Gamow representation is adequate.

# 2 Gamow States

Following [1, 2, 3] we define a Gamow-state in a dispersion-less space as

$$|\psi_G\rangle = \int_{-\infty}^{\infty} \{\mathcal{H}[\mathcal{I}(p)]\mathcal{H}(x) - \mathcal{H}[-\mathcal{I}(p)]\mathcal{H}(-x)\} e^{\frac{ipx}{\hbar}} |x\rangle dx, \quad (2.1)$$

or

$$\psi_G(x) = \{\mathcal{H}[\mathcal{I}(p)]\mathcal{H}(x) - \mathcal{H}[-\mathcal{I}(p)]\mathcal{H}(-x)\} e^{\frac{ipx}{\hbar}}. \quad (2.2)$$

The norm-squared for such a state reads

$$\langle \psi_G | \psi_G \rangle = \int_0^\infty \mathcal{H}[\Im(p)] e^{\frac{i(p-p^*)x}{\hbar}} dx - \int_{-\infty}^0 \mathcal{H}[-\Im(p)] e^{\frac{i(p-p^*)x}{\hbar}} dx. \quad (2.3)$$

These integrals can be easily evaluated. One finds

$$\langle \psi_G | \psi_G \rangle = \{\mathcal{H}[\Im(p)] - \mathcal{H}[-\Im(p)]\} \frac{\hbar}{i(p^* - p)} = \frac{\hbar}{2|\Im(p)|}. \quad (2.4)$$

Accordingly, the normalized Gamow-state  $\phi_G$  becomes

$$|\phi_G \rangle = \sqrt{\frac{2|\Im(p)|}{\hbar}} |\psi_G \rangle. \quad (2.5)$$

Since

$$\langle \phi_G | (H |\phi_G \rangle) = \frac{p^2}{2m}, \quad (2.6)$$

$$(\langle \phi_G | H) |\phi_G \rangle = \frac{p^{*2}}{2m}, \quad (2.7)$$

one encounters, for the energy mean value [1, 2, 3]

$$\langle H \rangle = \frac{1}{2} [\langle \phi_G | (H |\phi_G \rangle) + (\langle \phi_G | H) |\phi_G \rangle] = \frac{p^2 + p^{*2}}{4m} = \frac{\Re(p^2)}{2m}. \quad (2.8)$$

in order to obtain the probability distribution associated to a q-Gamow state we start by looking at the scalar product between this state and a free one:

$$\langle \phi | \phi_G \rangle = \frac{1}{\hbar} \sqrt{\frac{|\Im(p)|}{\pi}} \left\{ \int_0^\infty \mathcal{H}[\Im(p)] e^{\frac{i(p-k)x}{\hbar}} dx - \int_{-\infty}^0 \mathcal{H}[-\Im(p)] e^{\frac{i(p-k)x}{\hbar}} dx \right\}. \quad (2.9)$$

Thus,

$$\langle \phi | \phi_G \rangle = \frac{i \sqrt{\frac{|\Im(p)|}{\pi}}}{p - k} \quad (2.10)$$

The ensuing probability distribution is the Breit-Wigner one

$$|\langle \phi | \phi_G \rangle|^2 = \frac{|\Im(p)|}{\pi \{[\Re(p) - k]^2 + \Im(p)^2\}}. \quad (2.11)$$

### 3 q-Gamow States

According to the q-statistics strictures (this word exists!) we must replace everywhere ordinary exponentials by so-called q-exponentials  $e_q(x)$  [11]

$$e_q(x) = [1 + (1 - q)x]^{1/(1-q)}; \quad q \in \mathcal{R}, \quad (3.1)$$

that becomes the ordinary exponential at  $q = 1$ . Accordingly,

$$|\psi_{qG}\rangle = \int_{-\infty}^{\infty} \{\mathcal{H}[\mathcal{I}(p)]\mathcal{H}(x) - \mathcal{H}[-\mathcal{I}(p)]\mathcal{H}(-x)\} \otimes \left[1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} |x\rangle dx, \quad (3.2)$$

or

$$\psi_{qG}(x) = \{\mathcal{H}[\mathcal{I}(p)]\mathcal{H}(x) - \mathcal{H}[-\mathcal{I}(p)]\mathcal{H}(-x)\} \left[1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}}. \quad (3.3)$$

The norm of a q-Gamow state is

$$\begin{aligned} \langle \psi_{qG} | \psi_{qG} \rangle &= \int_0^{\infty} \mathcal{H}[\mathcal{I}(p)] \left[1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} \left[1 + \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} dx \\ &+ \int_{-\infty}^0 \mathcal{H}[-\mathcal{I}(p)] \left[1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} \left[1 + \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} dx, \end{aligned} \quad (3.4)$$

or equivalently,

$$\begin{aligned} \langle \psi_{qG} | \psi_{qG} \rangle &= \int_0^{\infty} \mathcal{H}[\mathcal{I}(p)] \left[1 + \frac{2(q-1)\mathcal{I}(p)x}{\hbar\sqrt{2(q+1)}} + \frac{(q-1)^2|p|^2x^2}{\hbar^2 2(q+1)}\right]^{\frac{2}{1-q}} dx \\ &+ \int_0^{\infty} \mathcal{H}[-\mathcal{I}(p)] \left[1 - \frac{2(q-1)\mathcal{I}(p)x}{\hbar\sqrt{2(q+1)}} + \frac{(q-1)^2|p|^2x^2}{\hbar^2 2(q+1)}\right]^{\frac{2}{1-q}} dx, \end{aligned} \quad (3.5)$$

that can be recast as

$$\langle \psi_{qG} | \psi_{qG} \rangle = \int_0^\infty \left[ 1 + \frac{2(q-1)|\Im(\mathbf{p})|x}{\hbar\sqrt{2(q+1)}} + \frac{(q-1)^2|p|^2x^2}{\hbar^2 2(q+1)} \right]^{\frac{2}{1-q}} dx. \quad (3.6)$$

We effect now the change of variables  $y = \frac{(q-1)|p|x}{\hbar\sqrt{2(q+1)}}$  and obtain

$$\langle \psi_{qG} | \psi_{qG} \rangle = \frac{\hbar\sqrt{2(q+1)}}{(q-1)|p|} \int_0^\infty \left[ 1 + \frac{2|\Im(\mathbf{p})|y}{|p|^2} + y^2 \right]^{\frac{2}{1-q}} dy. \quad (3.7)$$

After a new change of variables  $z = y + \frac{|\Im(\mathbf{p})|}{|p|}$  we find

$$\langle \psi_{qG} | \psi_{qG} \rangle = \frac{\hbar\sqrt{2(q+1)}}{(q-1)|p|} \int_{\frac{|\Im(\mathbf{p})|}{|p|}}^\infty \left\{ z^2 + \frac{[\Re(\mathbf{p})]^2}{|p|^2} \right\}^{\frac{2}{1-q}} dz. \quad (3.8)$$

Finally, after a third change of variables  $s = z^2$  we get for our norm

$$\langle \psi_{qG} | \psi_{qG} \rangle = \frac{\hbar\sqrt{2(q+1)}}{2(q-1)|p|} \int_{\frac{[\Im(\mathbf{p})]^2}{|p|^2}}^\infty s^{-\frac{1}{2}} \left\{ s + \frac{[\Re(\mathbf{p})]^2}{|p|^2} \right\}^{\frac{2}{1-q}} ds. \quad (3.9)$$

Using the result given in [15] we arrive to:

$$\begin{aligned} \langle \psi_{qG} | \psi_{qG} \rangle &= \frac{\hbar}{5-q} \frac{\sqrt{2(q+1)}}{|p|} \left\{ \frac{[\Im(\mathbf{p})]^2}{|p|^2} \right\}^{\frac{q-5}{2(q-1)}} \otimes \\ &\quad F\left(\frac{2}{q-1}, \frac{5-q}{2(q-1)}, \frac{3+q}{2(q-1)}, -\frac{[\Re(\mathbf{p})]^2}{[\Im(\mathbf{p})]^2}\right). \end{aligned} \quad (3.10)$$

It is shown in [14] that

$$\begin{aligned} &F\left(\frac{2}{q-1}, \frac{5-q}{2(q-1)}, \frac{3+q}{2(q-1)}, -\frac{[\Re(\mathbf{p})]^2}{[\Im(\mathbf{p})]^2}\right) = \\ &\left\{ \frac{|p|^2}{[\Im(\mathbf{p})]^2} \right\}^{\frac{q-5}{2(q-1)}} F\left(\frac{1}{2}, \frac{5-q}{2(q-1)}, \frac{3+q}{2(q-1)}, \frac{[\Re(\mathbf{p})]^2}{|p|^2}\right), \end{aligned} \quad (3.11)$$

which yields for the norm the expression

$$\begin{aligned} \langle \psi_{qG} | \psi_{qG} \rangle &= \frac{\hbar}{5-q} \frac{\sqrt{2(q+1)}}{|p|} \otimes \\ &F\left(\frac{1}{2}, \frac{5-q}{2(q-1)}, \frac{3+q}{2(q-1)}, \frac{[\Re(p)]^2}{|p|^2}\right) = [A(q, p)]^2, \end{aligned} \quad (3.12)$$

so that the normalized  $q$ -Gamow state becomes

$$|\phi_{qG}\rangle = [A(q, p)]^{-1} |\psi_{qG}\rangle. \quad (3.13)$$

Noticing that

$$\lim_{q \rightarrow 1} F\left(\frac{1}{2}, \frac{5-q}{2(q-1)}, \frac{3+q}{2(q-1)}, \frac{[\Re(p)]^2}{|p|^2}\right) = F\left(\frac{1}{2}, 4; 4; \frac{[\Re(p)]^2}{|p|^2}\right) \quad (3.14)$$

and using a result of [16] one has

$$F\left(\frac{1}{2}, 4; 4; \frac{[\Re(p)]^2}{|p|^2}\right) = \left[\frac{[\Im(p)]^2}{|p|^2}\right]^{-\frac{1}{2}}, \quad (3.15)$$

and

$$\lim_{q \rightarrow 1} [A(q, p)]^2 = \frac{\hbar}{2|\Im(p)|}. \quad (3.16)$$

Using now, from [17],

$$H\phi_q(x) = \frac{p^2}{2m} [\phi_q(x)]^q, \quad (3.17)$$

we encounter

$$\begin{aligned} \langle \phi_{qG} | (H | \phi_{qG} \rangle) &= [A(p, q)]^{-2} \frac{p^2}{2m} \otimes \\ &\left\{ \int_0^\infty \mathcal{H}[\Im(p)] \left[1 - \frac{i(q-1)p x}{\hbar \sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}} \left[1 + \frac{i(q-1)p^* x}{\hbar \sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} dx \right. \\ &\left. + \int_{-\infty}^0 \mathcal{H}[-\Im(p)] \left[1 - \frac{i(q-1)p x}{\hbar \sqrt{2(q+1)}}\right]^{\frac{2q}{1-q}} \left[1 + \frac{i(q-1)p^* x}{\hbar \sqrt{2(q+1)}}\right]^{\frac{2}{1-q}} dx \right\}, \end{aligned} \quad (3.18)$$

or equivalently,

$$\begin{aligned}
\langle \phi_{qG} | (H | \phi_{qG} \rangle) &= [A(p, q)]^{-2} \frac{p^2}{2m} \otimes \\
&\left\{ \int_0^\infty \mathcal{H}[\mathcal{I}(p)] \left[ 1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} \left[ 1 + \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} dx \right. \\
&\left. + \int_0^\infty \mathcal{H}[-\mathcal{I}(p)] \left[ 1 + \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} \left[ 1 - \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} dx \right\}. \quad (3.19)
\end{aligned}$$

We can recast (3.19) as

$$\begin{aligned}
\langle \phi_{qG} | (H | \phi_{qG} \rangle) &= [A(p, q)]^{-2} \frac{p^2}{2m} \otimes \\
&\left\{ \mathcal{H}[\mathcal{I}(p)] \left[ \frac{i(q-1)p^*}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} \left[ \frac{-i(q-1)p}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} \otimes \right. \\
&\int_0^\infty \left[ 1 - \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} \left[ 1 + \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} dx \\
&+ \mathcal{H}[-\mathcal{I}(p)] \left[ \frac{-i(q-1)p^*}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} \left[ \frac{i(q-1)p}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} \otimes \\
&\left. \int_0^\infty \left[ 1 + \frac{i(q-1)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2q}{1-q}} \left[ 1 - \frac{i(q-1)p^*x}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} dx \right\}. \quad (3.20)
\end{aligned}$$

We use now a result from [18] and obtain

$$\begin{aligned}
\langle \phi_{qG} | (H | \phi_{qG} \rangle) &= -\frac{\hbar}{i[A(p, q)]^2} \frac{p}{2m} \mathcal{B} \left( 1, \frac{3+q}{q-1} \right) \frac{\sqrt{2(q+1)}}{(q-1)} \otimes \\
&\{\mathcal{H}[\mathcal{I}(p)] - \mathcal{H}[-\mathcal{I}(p)]\} F \left( 1, \frac{2}{q-1}; \frac{2(q+1)}{q-1}; 1 + \frac{p^*}{p} \right), \quad (3.21)
\end{aligned}$$

or equivalently,

$$\begin{aligned} \langle \Phi_{qG} | (H | \Phi_{qG} \rangle) &= -\frac{\hbar}{i[A(p, q)]^2} \frac{p}{2m} \frac{\sqrt{2(q+1)}}{(3+q)} \otimes \\ &\{ \mathcal{H}[\mathfrak{I}(p)] - \mathcal{H}[-\mathfrak{I}(p)] \} F \left( 1, \frac{2}{q-1}; \frac{2(q+1)}{q-1}; 1 + \frac{p}{p^*} \right). \end{aligned} \quad (3.22)$$

In analogous fashion we find

$$\begin{aligned} (\langle \Phi_{qG} | H) | \Phi_{qG} \rangle &= \frac{\hbar}{i[A(p, q)]^2} \frac{p^*}{2m} \frac{\sqrt{2(q+1)}}{(3+q)} \otimes \\ &\{ \mathcal{H}[\mathfrak{I}(p)] - \mathcal{H}[-\mathfrak{I}(p)] \} F \left( 1, \frac{2}{q-1}; \frac{2(q+1)}{q-1}; 1 + \frac{p}{p^*} \right). \end{aligned} \quad (3.23)$$

Thus, according to [1, 2, 3] we obtain for the mean energy value

$$\langle H \rangle_q = \frac{1}{2} [\langle \Phi_{qG} | (H | \Phi_{qG} \rangle) + (\langle \Phi_{qG} | H) | \Phi_{qG} \rangle]. \quad (3.24)$$

Additionally, since

$$\lim_{q \rightarrow 1} F \left( 1, \frac{2}{q-1}; \frac{2(q+1)}{q-1}; 1 + \frac{p}{p^*} \right) = \frac{2p}{p - p^*}, \quad (3.25)$$

we have

$$\lim_{q \rightarrow 1} \langle H \rangle_q = \frac{\Re(p^2)}{2m} = \langle H \rangle. \quad (3.26)$$

We investigate now the  $q$ -analogue of the Breit-Wigner distribution tackling

$$\begin{aligned} \langle \Phi | \Phi_{Gq} \rangle &= \frac{1}{\sqrt{2\pi\hbar A(q, p)}} \left\{ \mathcal{H}[\mathfrak{I}(p)] \int_0^\infty e^{-ikx} \left[ 1 + \frac{i(1-q)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} dx \right. \\ &\quad \left. - \mathcal{H}[-\mathfrak{I}(p)] \int_0^\infty e^{ikx} \left[ 1 - \frac{i(1-q)px}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} dx \right\}, \end{aligned} \quad (3.27)$$



and rewrite it as

$$\begin{aligned}
\langle \phi | \phi_{Gq} \rangle = & \frac{1}{\sqrt{2\pi\hbar A(q, p)}} \left\{ H[\mathfrak{I}(p)] \left[ \frac{i(1-q)p}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} \otimes \right. \\
& \int_0^\infty e^{-ikx} \left[ x + \frac{\hbar\sqrt{2(q+1)}}{i(1-q)p} \right]^{\frac{2}{1-q}} dx - H[-\mathfrak{I}(p)] \left[ -\frac{i(1-q)p}{\hbar\sqrt{2(q+1)}} \right]^{\frac{2}{1-q}} \otimes \\
& \left. \int_0^\infty e^{ikx} \left[ x - \frac{\hbar\sqrt{2(q+1)}}{i(1-q)p} \right]^{\frac{2}{1-q}} dx \right\}. \quad (3.28)
\end{aligned}$$

We appeal now to a result of [19] and obtain

$$\begin{aligned}
\langle \phi | \phi_{Gq} \rangle = & -i \sqrt{\frac{\hbar}{2\pi A(q, p)}} \left[ \frac{\sqrt{2(q+1)}}{(1-q)p} \right]^{\frac{2}{q-1}} k^{\frac{3-q}{q-1}} e^{\frac{\sqrt{2(q+1)}k}{(1-q)p}} \otimes \\
& \Gamma \left[ \frac{3-q}{1-q}, \frac{\sqrt{2(q+1)}k}{(1-q)p} \right], \quad (3.29)
\end{aligned}$$

which leads to the q-Breit-Wigner result

$$\begin{aligned}
|\langle \phi | \phi_{Gq} \rangle|^2 = & \frac{\hbar}{2\pi A(q, p)} \left[ \frac{2(q+1)}{(1-q)^2 |p|^2} \right]^{\frac{2}{q-1}} k^{\frac{2(3-q)}{q-1}} e^{\frac{\sqrt{2(q+1)}k(p+p^*)}{(1-q)|p|^2}} \otimes \\
& \Gamma \left[ \frac{3-q}{1-q}, \frac{\sqrt{2(q+1)}k}{(1-q)p} \right] \left\{ \Gamma \left[ \frac{3-q}{1-q}, \frac{\sqrt{2(q+1)}k}{(1-q)p} \right] \right\}^*. \quad (3.30)
\end{aligned}$$

Note that (3.27) converges uniformly for  $q \rightarrow 1$  [since the q-exponential converges in that way to the ordinary one], i.e.,

$$\lim_{q \rightarrow 1} |\langle \phi | \phi_{Gq} \rangle|^2 = \frac{|\mathfrak{I}(p)|}{\pi \{[\mathfrak{R}(p) - k]^2 + \mathfrak{I}(p)^2\}}. \quad (3.31)$$

## 4 Conclusions

In this work we have introduced q-Gamow states. For that purpose we have computed their norm, the mean energy value, and the concomitant q-Breit-Wigner distributions. In all instance, results tend to the customary ones for  $q \rightarrow 1$ .

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